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Evidence for a Cosmological Phase Transition on the TeV Scale*

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Abstract

Examining the reverse evolution of the universe from the present, long before reaching Planck density dynamics one expects major modifications from the de-coherent thermal equations of state, suggesting a prior phase that has macroscopic coherence properties. The assumption that the phase transition occurs during the radiation dominated epoch, and that zero-point motions drive the fluctuations associated with this transition, specifies a class of cosmological models in which the cosmic microwave background fluctuation amplitude at last scattering is approximately 10^{-5} . Quantum measurability constraints (eg. uncertainty relations) define cosmological scales whose expansion rates can be at most luminal. Examination of these constraints for the observed dark energy density establishes a time interval from the transition to the present. It is shown that the dark energy can consistently be interpreted as due to the vacuum energy of collective gravitational modes which manifest as the zero-point motions of coherent Planck scale mass units prior to the gravitational quantum decoherence of the cosmology. A scenario is suggested that connects microscopic physics to the relevant cosmological scale.

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1 Introduction

There is general (although not universal) agreement among physical cosmologists that the current expansion phase in the evolution of our universe can be extrapolated back toward an initial state of compression so extreme that we can neither have direct laboratory nor indirect (astronomical) observational evidence for the laws of physics needed to continue that extrapolation. However, much can be ascertained about the physics that is transmitted through the expansion by the evidence available to modern astrophysical measurements. For an excellent review of the status of observational cosmology, see reference [1].

Here, it is assumed that the experimental evidence for currently accepted theories of particle physics is relevant in the TeV range. It is further assumed that the current understanding of general relativity as a gravitational theory is adequate over the same range, and consequently that the cosmological Friedman-Lemaitre (FL) dynamical equations are reliable guides once the observational regime has been reached where the homogeneity and isotropy assumptions on which those equations are based become consistent with astronomical data to requisite accuracy. The elementary particle theories usually employed in relativistic quantum field theories have well defined transformation properties in the flat Minkowski space of special relativity; their fundamental principles are assumed to apply on coordinate backgrounds with cosmological curvature. There is direct experimental evidence that quantum mechanics does apply in the background space provided by the Schwarzschild metric of the Earth from experiments by Overhauser and collaborators[2, 3]. These experiments show that coherent self-interference of single neutrons changes as expected when the plane of the two interfering paths is rotated from being parallel to being perpendicular to the “gravitational field” of the Earth. These were the first measurements requiring both Newton’s constant and Planck’s constant. They provide a verification of the principle of equivalence for quantum systems.

It is therefore expected that during some period in the past, quantum coherence of gravitating systems should have qualitatively altered the thermodynamics of the cosmology. Often, the onset of the importance of quantum effects in gravitation is

taken to be at the Planck scale. However, as is the case with Fermi degenerate stars, this need not be true of the cosmology as a whole. Quantum coherence refers to the entangled nature of quantum states for space-like separations. This is made evident by superluminal correlations (without the exchange of signals) in the observable behavior of such quantum states. Note that the exhibition of quantum coherent behavior for gravitating systems does *not* require the quantization of the gravitation field.

The (luminal) horizon problem for present day cosmology arises from the large scale homogeneity and isotropy of the observed universe. Examining the ratio of the present conformal time η_o (which multiplied by c is the distance a photon can travel in a given time) with that during recombination η_* , $\frac{\eta_o}{\eta_*} \sim 100$, the subsequent expansion is expected to imply that light from the cosmic microwave background would come from $100^3 = 10^6$ luminally disconnected regions. Yet, angular correlations of the fluctuations across the whole sky have been accurately measured by several experiments[7]. The observation of these correlations provides evidence for a space-like coherent phase associated with the cosmological fluctuations that produced the Cosmic Microwave Background (CMB).

The approach taken here will not depend on the present particle horizon scale, which is an accident of history. It will be argued that the equilibration of microscopic interactions can only occur on cosmological scales consistent with quantum measurement constraints[4]. Global quantum coherence on larger scales solves the horizon problem, since quantum correlations are in this sense supraluminal.

The luminosities of distant Type Ia supernovae show that the rate of expansion of the universe has been accelerating for several giga-years[5]. This conclusion is independently confirmed by analysis of the Cosmic Microwave Background (CMB) radiation[6, 7]. Both results are in quantitative agreement with a (positive) cosmological constant fit to the data. The existence of a cosmological constant / dark energy density defines a length scale that should be consistent with those scales generated by the microscopic physics, and must be incorporated in any description of the evolution of the universe. When the dynamics of the cosmology is made consistent with this scale, it is expected that energy scales associated with the usual microscopic interactions of relativistic quantum mechanics (QED, QCD, etc) cannot contribute to

cosmological (gravitational) affects when the relevant Friedmann-Robertson-Walker (FRW) scale expansion rate is supra-luminal[8], $\dot{R}_\epsilon > c$. Quantum measurement arguments will be made below for quantized energy units ϵ requiring that the scales associated with those energies, R_ϵ , must satisfy $\dot{R}_\epsilon \leq c$. The gravitationally coherent cosmological dark energy density should decouple from the thermalized energy density in the Friedmann-Lemaître(FL) equations when the relevant FRW scale expansion rate is no longer supra-luminal. A key assumption in this paper is that this decoupling corresponds to a phase transition from some form of a macroscopic coherent state to a state understood in terms of late time observations. The form of the FL equations 4.1 and 4.2 is clearly scale invariant in R (other than the term involving spatial curvature) for given intensive densities, which might lead one to question the physical meaning of R . However, this scale invariance does *not* negate an assignment of a meaningful FRW scale parameter whose dynamics is governed by this equation. The scale below which the cosmology is no longer homogeneous is an example of a well defined particular scale of cosmological significance whose evolution is governed by the interplay of the dynamics of the FL equations with the density perturbations. Likewise, any infrared cutoff wavelength associated with the physics that generates those perturbations is an example of such a scale.

The basic assumption in this paper is that the quantum zero-point energies whose effects on the subsequent cosmology are fixed by the phase transition mentioned in the previous paragraph are to be identified with the cosmological constant, or “dark energy”. It has been shown elsewhere[9] that the expected amplitude of fluctuations driven by dark energy as the energy density de-coheres is of the order needed to evolve into the fluctuations observed in CMB radiation and in galactic clustering. From the time of this phase transition until this scale factor expansion rate again becomes supra-luminal due to the cosmological constant, the usual expansion rate evolution predicted by FL dynamics, including a cosmological constant, is expected to hold.

Since this paper will eventually identify “dark energy” as a particular “vacuum energy” driven by zero-point motions, it is worthwhile to examine the physics behind other systems that manifest vacuum energy. One physical system in which vacuum energy density directly manifests is the Casimir effect[10]. Casimir considered the

change in the vacuum energy due to the placement of two parallel plates separated by a distance a . He calculated an energy per unit area of the form

$$\frac{\frac{1}{2}(\sum_{modes} \hbar c k_{plates} - \sum_{modes} \hbar c k_{vacuum})}{A} = -\frac{\pi^2 \hbar c}{720 a^3} \quad (1.1)$$

resulting in an attractive force of given by

$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4} \cong \frac{-0.013 \text{ dynes}}{(a/\text{micron})^4} \text{ cm}^{-2}, \quad (1.2)$$

independent of the charges of the sources. Although the effect does not depend on the electromagnetic coupling strength of the sources, it does depend on the nature of the interaction and configuration. Lifshitz and his collaborators[11] demonstrated that the Casimir force can be thought of as the superposition of the van der Waals attractions between individual molecules that make up the attracting media. This allows the Casimir effect to be interpreted in terms of the zero-point motions of the sources as an alternative to vacuum energy. At zero temperature, the coherent zero-point motions of source currents on opposing plates correlate in a manner resulting in a net attraction, whereas if the motions were independently random, there would be no net attraction. On dimensional grounds, one can determine that the number of particles per unit area undergoing zero-point motions that contribute to the Casimir result vary as a^{-2} . Boyer[12] and others subsequently derived a repulsive force for a spherical geometry of the form

$$\frac{1}{2} \left(\sum_{modes} \hbar c k_{sphere} - \sum_{modes} \hbar c k_{vacuum} \right) = \frac{0.92353 \hbar c}{a}. \quad (1.3)$$

This shows that the change in electromagnetic vacuum energy *is* dependent upon the geometry of the boundary conditions. Both predictions have been confirmed experimentally. It is important to note that this energy grows inversely with the geometrical scale $E_{Casimir} \sim \frac{\hbar c}{a}$.

Generally, correlated zero-point motions can be used to describe vacuum energy effects in the Casimir effect. As expressed by Daniel Kleppner[13],

The van der Waals interaction is generally described in terms of a correlation between the instantaneous dipoles of two atoms or molecules.

However, it is evident that one can just as easily portray it as the result of a change in vacuum energy due to an alteration in the mode structure of the system. The two descriptions, though they appear to have nothing in common, are both correct.

Additionally, as pointed out by Wheeler and Feynmann[14], and others[15], one cannot unambiguously separate the properties of fields from the interaction of those fields with their sources and sinks. Since there are no manifest boundaries in the description of early cosmology presented here, it is much more convenient to examine the effects of any gravitational “vacuum energy” in terms of the correlated zero-point motions of the sources of those gravitational fields. In what follows, the zero-point motions of coherent sources will be considered to correspond to the vacuum energies of the associated quanta. This point is elaborated at the start of Sec. 2

The introduction of energy density ρ into Einstein’s equation introduces a preferred rest frame with respect to any normal energy density. However, as can be seen in the Casimir effect, the vacuum need not exhibit velocity dependent effects which would break Lorentz invariance. Although a single moving mirror has been predicted to experience dissipative effects from the vacuum due to its motion, these effects can be shown to be of 5th order in time derivatives of position[16].

Another system which manifests physically measurable effects due to zero-point energy is liquid ^4He . One sees that this is the case by noting that atomic radii are related to atomic volume V_a (which can be measured) by $R_a \sim V_a^{1/3}$. The uncertainty relation gives momenta of the order $\Delta p \sim \hbar/V_a^{1/3}$. Since the system is non-relativistic, one estimates the zero-point kinetic energy to be of the order $E_o \sim \frac{(\Delta p)^2}{2m_{He}} \sim \frac{\hbar^2}{2m_{He}V_a^{2/3}}$. The minimum in the potential energy is located around R_a , and because of the low mass of ^4He , the value of the small attractive potential is comparable to the zero-point kinetic energy. Therefore, this bosonic system forms a low density liquid. The lattice spacing for solid helium would be expected to be even smaller than the average spacing for the liquid. This means that a large external pressure is necessary to overcome the zero-point energy in order to form solid helium. Generally zero-point motions correspond to the saturation of the triangle inequality as expressed in the uncertainty

principle.

Applying this reasoning to relativistic gravitating mass units with quantum coherence within the volume generated by a Compton wavelength λ_m^3 , the zero point momentum is expected to be of order $p \sim \frac{\hbar}{V^{1/3}} \sim \frac{\hbar}{\lambda_m}$. This gives a zero point energy of order $E_0 \approx \sqrt{2}mc^2$. If we estimate a mean field potential from the Newtonian form $V \sim -\frac{G_N m^2}{\lambda_m} = -\frac{m^2}{M_P^2} mc^2 \ll E_0$, it is evident that the zero point energy will dominate the energy of such a system.

As a further illustration of the expectation of the manifestation of macroscopic quantum effects on a cosmological scale, consider particulate dark matter. If the dark matter behaves as a bosonic particle, one can estimate the critical number density by examining a free bose gas. For non-relativistic dark matter, this critical density is reached when thermal modes can no longer accomodate a distribution of all of the particles, forcing macroscopic occupation of the lowest energy state. For particles of mass m , this density satisfies[17]

$$\frac{N_{DM}}{V} = \frac{\zeta(3/2)\Gamma(3/2)}{(2\pi)^2\hbar^3} (2mk_B T_{crit})^{3/2} , \quad \rho_{DM} \cong \frac{N_{DM}}{V} mc^2. \quad (1.4)$$

This equation can be used to determine the relationship between the critical temperature and the redshift associated with Bose condensation of such particles, given by

$$k_B T_{crit} = \frac{1}{2} \frac{\rho_{DM}^{2/3}}{(mc^2)^{5/3}} \frac{(2\pi)^2(\hbar c)^3}{\zeta(3/2)\Gamma(3/2)} \approx 1.7 \times 10^{-31} GeV \frac{z^2}{g_m^{2/3}} \left(\frac{GeV}{mc^2} \right)^{5/3}. \quad (1.5)$$

The temperature of the photon gas is expected to scale with the redshift when the appropriate pair creation threshold effects are properly incorporated. Setting the critical temperature the same as the photon temperature gives an estimate of the onset of cosmological bose condensation for such particles, given by

$$z_{crit} \cong 1.4 \times 10^{18} \left(\frac{g(T_{\gamma o})}{g(T_{crit})} \right)^{1/3} g_m^{2/3} \left(\frac{mc^2}{GeV} \right)^{5/3}, \quad (1.6)$$

where $g(z)$ counts the number of low mass degrees of freedom available at redshift z . The transition occurs at non-relativistic energies as long as the particulate mass satisfies $m > \frac{15eV}{g_m}$. Some authors have suggested that present day Bose condensation addresses some of the problems associated with the dark matter[18].

Adiabatic expansion is expected to preserve the ratio of particulate dark matter particle number to photon number. Relating photon energy density to number density gives

$$\frac{N_{DM}}{N_\gamma} = \frac{\zeta(4)\Gamma(4)}{\zeta(3)\Gamma(3)} \left(\frac{\Omega_{DMo}}{\Omega_{\gamma o}} \right) \frac{k_B T_{\gamma o}}{mc^2} \cong 3.7 \times 10^{-9} \left(\frac{GeV}{m_{DM}c^2} \right). \quad (1.7)$$

This represents a phenomenological handle on any cosmological affects due to thermalization of particulate bose condensed dark matter.

The Friedman-Robertson-Walker (FRW) metric for a homogeneous isotropic cosmology is given by

$$ds^2 = c^2 dt^2 + R^2(t) \left(\frac{dr^2}{1 - k r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \quad (1.8)$$

In a radiation-dominated universe this backward extrapolation in time (which taken literally *must* terminate when *any* FRW scale factor $R(t)$ goes to zero and its time rate of change $\dot{R}(t)$ goes to infinity) is guaranteed to reach the speed of light $\dot{R}(t_c) = c$ at some finite time t_c when the scale factor $R(t_c)$ still has a small, but finite, value. Here the FRW scale factor R is taken to have dimensions of length (not to be the dimensionless scale relative to the present horizon $a(t) \equiv R(t)/R_o$), with dr being dimensionless. One aim of this paper is to show that for quantized energies, a microscopically motivated cosmological scale can be defined which indicates a macroscopic phase transition after which that scale expands at a sub-luminal rate.

The general approach used here has been to start from well understood macro-physics, assume that the physics of a cosmological phase transition defines an FRW scale parameter, and examine cosmological physics at the time when the physical consistency of the thermal state of the cosmology is called into question. For times after that transition there is general confidence that well understood micro- *and* macro-physics are valid at the cosmological level. The relevant FRW scale parameter must be expressed in terms of the scales of microscopic physics. The calculations presented here will not use the present horizon (Hubble) scale other than to evaluate observed phenomenological parameters.

The “horizon problem” addresses the extreme uniformity of the Cosmic Microwave Background Radiation across multitudes of space-like separated regions, and the

space-like correlation in the phase of the fluctuation across the entire sky, despite these regions being luminally disconnected at the time of last scattering, when these photons were produced. In quantum physics, such phenomena are not unusual. Using the usual vacuum state in Minkowski space-time, the equal time correlation function $\langle vac|\Psi(x, y, z, t)\Psi(x', y', z', t)|vac\rangle$ of a quantum field $\Psi(\vec{x})$ does not vanish for space-like separations. For example, for massless scalar fields, $\langle vac|\Psi(\vec{x})\Psi(\vec{y}) + \Psi(\vec{y})\Psi(\vec{x})|vac\rangle = \frac{1}{4\pi^2 s^2}$, where the proper distance satisfies $s^2 = |\mathbf{x} - \mathbf{y}|^2 - (x^0 - y^0)^2$, which falls off with the inverse square of the distance between the points. Since the vacuum expectation value of the field Ψ vanishes in the usual case, this clearly requires space-like correlations, ie

$$\begin{aligned} \frac{1}{4\pi^2 s^2} &= \langle vac|\Psi(\vec{x})\Psi(\vec{y}) + \Psi(\vec{y})\Psi(\vec{x})|vac\rangle \neq \\ 2 \langle vac|\Psi(\vec{x})|vac\rangle &< vac|\Psi(\vec{y})|vac\rangle = 0. \end{aligned} \quad (1.9)$$

However, since the commutator of the field *does* vanish for space-like separations, a measurement at \vec{y} cannot change the probability distribution at \vec{x} . In the approach here taken, global gravitational coherence solves (or defers) the horizon problem because the gravitational correlations implied by the FL equations *are* space-like; it is hypothesized that the same will be true of any type of dark energy of gravitational origin to be considered.

2 Estimate of Density Fluctuations

As has been argued, vacuum energy can often be thought of as resulting from the zero-point motions of the sources[11], and this is expected to be true of gravitational interactions. This argument is supported by the calculation of Bohr and Rosenfeld[19], who minimized the effect of a classical measurement of an electric field averaged over a finite volume on the value of a magnetic field at right angles averaged over (i) a non-overlapping volume, and (ii) an overlapping volume, and *vice versa*. When this minimum disturbance is put equal to the minimum uncertainty which the uncertainty principle allows for the measurement they reproduce the result of averaging the quantum mechanical commutation relations over the corresponding volumes.

Such arguments can be extended to the corresponding case when the sources and detectors are gravitational.

As the zero-point motions de-cohere and become localized, the deviations from uniformity are expected to appear as fluctuations in the cosmological energy density. Since these motions are inherently a quantum effect, one expects the fluctuations to exhibit the space-like correlations consistent with a quantum phenomenon. Measurable effects of quantum mechanical de-coherence are expected to manifest stochastically.

As an intuitive guide into how dark energy might freeze out as a system decoheres, consider a uniform distribution of non-relativistic masses M interacting pairwise through simple harmonic potentials at zero temperature. If charges were used as field sources rather than masses, as explained above the uncertainty relations associated with the positions and momenta of the charges result in averaged commutation relations between the electric and magnetic fields which are classically produced by those charges. Zero-point motions of the oscillators give equal partitioning of energies $\frac{1}{4}\hbar\omega$ to the kinetic and potential components. The correlations of motions of the masses do not vanish, although time averages do vanish. If the masses evaporate, the kinetic component is expected to drive density fluctuations of an expanding gas, whereas the potential components remain in the springs (as zero point tensions) of fixed density, as illustrated in Figure 1. The compressional energy is expected to be frozen in as dark energy in the background during the phase transition on scales larger than the de-coherence scale. The mass units which were undergoing zero-point motions no longer behave as coherent masses after evaporation, and only reflect their former state through the density inhomogeneities generated during the evaporation process, and no longer couple to the frozen-in potential component of the zero point energy. Such an interpretation would clearly represent the source of a cosmological constant as a frozen energy density, *not* a fixed background metric variation.

An interacting sea of the quantum fluctuations due to zero point motions should exhibit local statistical variations in the energy. For a sufficiently well defined phase state, one should be able to use counting arguments to quantify these variations[9, 20]. The dark energy E_Λ is expected to have uniform density, and to drive fluctuations of



Figure 1: Early quantum stage undergoing zero-point fluctuations during decoherence, with evaporation and subsequent expansion of thermal energy density associated with masses which were previously coherently attached via the springs. Potential energy density gets frozen in during evaporation, whereas kinetic energy density drives density fluctuations.

the order

$$\langle (\delta E)^2 \rangle = E_\Lambda^2 \frac{d}{dE_\Lambda} \langle E \rangle \quad (2.1)$$

Given an equation of state $\langle E \rangle \sim (E_\Lambda)^b$, the expected fluctuations satisfy

$$\frac{\langle (\delta E)^2 \rangle}{\langle E \rangle^2} = b \frac{E_\Lambda}{\langle E \rangle}. \quad (2.2)$$

In terms of the densities, one can directly write $\frac{\langle (\delta E)^2 \rangle}{\langle E \rangle^2} = \frac{\langle (\delta \rho)^2 \rangle}{\rho^2} = b \frac{\rho_\Lambda}{\rho}$. At the time of the formation of the fluctuations, this means that the amplitude $\delta \rho / \rho$ is expected to be of the order

$$\delta_{PT} = \left(b \frac{\rho_\Lambda}{\rho_{PT}} \right)^{1/2} = \sqrt{b} \frac{R_{PT}}{R_\Lambda} \quad (2.3)$$

where ρ_{PT} is the cosmological energy density at the time of the phase transition that decouples the dark energy and $\Lambda = 8\pi G_N \rho_\Lambda / c^4 = 3/R_\Lambda^2$ is the cosmological constant.

For adiabatic perturbations (those that fractionally perturb the number densities of photons and matter equally), the energy density fluctuations grow according to[6]

$$\delta = \begin{cases} \delta_{PT} \left(\frac{R(t)}{R_{PT}} \right)^2 & \text{radiation-dominated} \\ \delta_{eq} \left(\frac{R(t)}{R_{eq}} \right) & \text{matter-dominated} \end{cases}, \quad (2.4)$$

which gives an estimate for the scale of fluctuations at last scattering from fluctuations

during the phase transition expressed by

$$\begin{aligned}\delta_{LS} &= \frac{(1+z_{PT})^2\sqrt{b}}{(1+z_{eq})(1+z_{LS})} \left(\frac{\rho_\Lambda}{\rho_{PT}} \right)^{1/2} \\ &\cong \frac{1}{1+z_{LS}} \sqrt{\frac{b\Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{eq})}} \cong 2.5 \times 10^{-5}\sqrt{b},\end{aligned}\tag{2.5}$$

where a spatially flat cosmology and radiation domination has been assumed. This estimate is independent of the density during the phase transition ρ_{PT} , and is of the order observed for the fluctuations in the CMB (see [6] section 23.2 page 221).

A tentative conclusion is that any *phenomenological* theory that accepts a constant dark energy density $\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$ as a reasonable way to fit the observational data indicating a flat, accelerating universe over the relevant range of red shifts, and which also assumes some sort of phase transition that decouples the residual dark energy from the subsequent dynamics of the energy density, will fall within a class of theories all of which fit the magnitude of the observed fluctuations.

3 Zero-Point Motions of Quantized Energy Scales

3.1 Gravitating quantum energies

For gravitating thermal systems, typical thermal energies $k_B T_{crit}$ are given by kinetic energies for constituent particles of mass m , which define a thermal distance scale $R_{thermal} \approx \frac{\hbar c}{k_B T_{crit}}$ that satisfies

$$R_{thermal} \lambda_m \sim (\Delta x)^2 \tag{3.1}$$

in terms of the Compton wavelength of the mass and the scale of zero-point motions of those masses. This relationship just follows from the momentum-space uncertainty principle. For example, for a degenerate free Fermi gas, the number density relationship $n = \frac{g_m}{6\pi^2} (2m\epsilon_{thermal})^{3/2}$ implies $R_\epsilon \lambda_m = \frac{1}{2} \left(\frac{6\pi^2}{g_m} \right)^{2/3} (\Delta x)^2$ [17].

If λ_E represents the microscopic coherence length of a correlated region of energy E , one expects a phase transition for densities of the order $\rho_{PT} \sim \frac{E}{\lambda_E^3}$. If densities in a region of gravitational coherence of scale R exceeds this value, one expects that

regions of quantum coherence of interaction energies of the order of E and scale λ_E will overlap sufficiently over the scale R_ϵ such that there will be a macroscopic quantum system on a cosmological scale. As long as the region of gravitational coherence is of

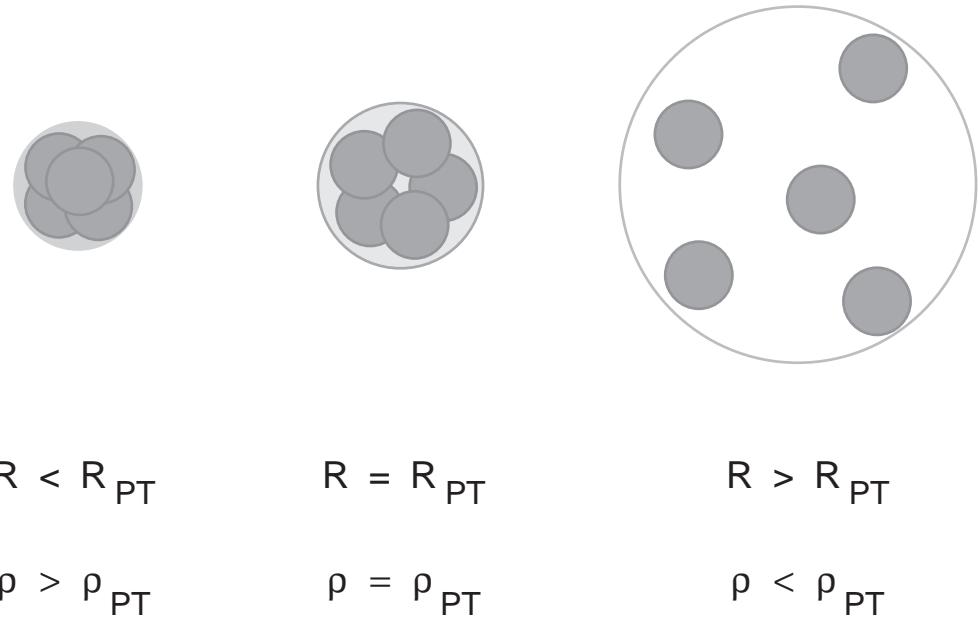


Figure 2: Overlapping regions of coherence during expansion

FRW scale $R \leq R_{PT}$, cosmological manifestations of this energy are of the scale R_{PT} . However, when the density of FL energy becomes less than ρ_{PT} , the coherence length of those microscopic energy units given by λ_E is insufficient to cover the cosmological scale, and the FL energy density will break into domains of cluster decomposed (see ref. [22] for scattering theoretical considerations) regions of local quantum coherence. This phase transition will decouple quantum coherence of gravitational interactions on the cosmological scale R_ϵ . At this stage (de-coherence), the cosmological (dark) vacuum energy density ρ_Λ associated with subtle gravitational coherence amongst the constituent energies is frozen at the scale determined by R_ϵ . The cosmological dark energy contribution to the expansion rate at that time is so small, and its coupling to de-coherent FL energy so insignificant, that its value is frozen at the value during

de-coherence given by

$$\rho_\Lambda \sim \frac{\epsilon}{R_\epsilon^3} = \frac{\epsilon^4}{(\hbar c)^3} \quad (3.2)$$

4 Gravitational De-coherence From Dark Energy

4.1 Quantum Measurability Constraints on Scale Expansion

Substitution of the isotropic, homogeneous FRW metric given by Eq. 1.8 into the Einstein Field equation driven by an ideal fluid result in the Friedmann-Lemaitre(FL) equations. The FL equations, which relate the rate and acceleration of the expansion to the fluid densities, are given by

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3c^2} (\rho + \rho_\Lambda) - \frac{kc^2}{R^2} \quad (4.1)$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3c^2} (\rho + 3P - 2\rho_\Lambda), \quad (4.2)$$

where $H(t)$ is the Hubble expansion rate, the dark energy density is given by $\rho_\Lambda = \frac{\Lambda c^4}{8\pi G_N}$, ρ represents the FL matter-energy density, and P is the pressure. The term which involves the spatial curvature k has explicit scale dependence on the FRW parameter R . The dark energy density is assumed to make a negligible contribution to the FL expansion during de-coherence, but will become significant as the FL energy density decreases due to the expansion of the universe.

If R is an arbitrary scale in the Friedmann-Lemaitre equations for a spatially flat space in the radiation-dominated epoch and a phase transition occurs at scale R_{PT} with expansion rate \dot{R}_{PT} , then Eq. 4.1 gives

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3c^4} \rho_{PT} \frac{R_{PT}^4}{R^4} = \frac{\dot{R}_{PT}^2 R_{PT}^2}{R^4} \quad (4.3)$$

which simplifies to the form

$$\dot{R}R = \dot{R}_{PT}R_{PT} = const. \quad (4.4)$$

This form can be integrated to obtain

$$t = \frac{R}{2\dot{R}} \quad (4.5)$$

where the conditions assume that $t = 0$ when $R = 0$, which smoothly defines the dynamics during this period. Since the approach used here involves a reverse extrapolation within the regime of known physics into a domain where that physics undergoes some type of phase transition or modification, the use of the analytic behavior of parameters in the radiation dominated regime is justified.

Any *quantized* energy scale ϵ defines a length scale R_ϵ by the relation

$$\epsilon = \frac{\hbar c}{R_\epsilon}. \quad (4.6)$$

Using a quantized energy scale of order ϵ and cosmological time associated with this scale via Eq. 4.5 in the energy-time uncertainty relation defines a constraint on the expansion rate associated with that scale:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow \frac{\hbar c}{R_\epsilon} \frac{R_\epsilon}{2\dot{R}_\epsilon} \geq \frac{\hbar}{2} \Rightarrow \dot{R}_\epsilon \leq c. \quad (4.7)$$

Therefore, assuming that the scale factor at the time of the phase transition is defined by a quantized microscopic energy scale $R_{PT} = R_\epsilon$, this scale must satisfy[21]

$$\dot{R}_{PT} = c. \quad (4.8)$$

That is, if it is required that ordinary, micoroscopic, event-decoherent *quantum* measurements satisfying the uncertainty principle are to be possible *after* the phase transition, *then* the rate of expansion of the FL scale factor associated with that phase transition must be sub-luminal. If the expansion rate associated with energy ϵ is supra-luminal $\dot{R}_\epsilon > c$, scattering states of this energy cannot form decomposed (decoherent) clusters of the type described in references [22, 23] on cosmological scales, i.e. incoherent decomposed clusters (scattering states) cannot be cosmologically formulated.

Setting the expansion rate to c in the Lemaitre equation 4.1 with $k = 0$, the energy density during dark energy de-coherence is given by

$$\rho_{PT} = \frac{3c^2}{8\pi G_N} \left(\frac{c}{R_{PT}} \right)^2 - \rho_\Lambda. \quad (4.9)$$

Similarly, the scale acceleration at the time of this transition can be determined:

$$\frac{\ddot{R}_{PT}}{R_{PT}} = -c^2 \left(\frac{1}{R_{PT}^2} + \frac{\Lambda}{3} \right) \Rightarrow \ddot{R}_{PT} \cong -\frac{c^2}{R_{PT}}. \quad (4.10)$$

Since the FL density at de-coherence is specified in terms of the single parameter given by the phase transition scale R_{PT} , all results which follow depend at most on this single parameter.

To estimate the time of the transition, if the behavior of the cosmology can be described as if it remains radiation dominated in the standard way down to $t = 0$, then the scale parameter satisfies

$$R(t) = R_{PT} \left(\frac{t}{t_{PT}} \right)^{1/2}, \quad (4.11)$$

which gives a time scale at de-coherence as

$$t_{PT} = \frac{R_{PT}}{2c}. \quad (4.12)$$

The assumption of radiation dominance during de-coherence corresponds to a thermal temperature of

$$g(T_{PT}) (k_B T_{PT})^4 \cong \frac{90}{8\pi^3} (M_P c^2)^2 \left(\frac{\hbar c}{R_{PT}} \right)^2, \quad (4.13)$$

where $g(T_{PT})$ counts the number of degrees of freedom associated with particles of mass $mc^2 \ll k_B T_{PT}$, and $M_P = \sqrt{\hbar c/G_N}$ is the Planck mass. Here we have used Eq. 4.9 and the energy density for relativistic thermal energy $\rho_{thermal} = g(T) \frac{\pi^2}{30} \frac{(k_B T)^4}{(\hbar c)^3}$.

4.2 Spatial Curvature Constraints

The energy density during dark energy de-coherence ρ_{PT} can be directly determined from the Lemaitre equation 4.1 to satisfy

$$H_{PT}^2 = \left(\frac{c}{R_{PT}} \right)^2 = \frac{8\pi G_N}{3c^2} (\rho_{PT} + \rho_\Lambda) - \frac{kc^2}{R_{PT}^2}. \quad (4.14)$$

A so called “open” universe ($k = -1$) is excluded from undergoing this transition, since the positive dark energy density term ρ_Λ already excludes a solution with $\dot{R} \leq c$. Likewise, for a “closed” universe that is initially radiation dominated, the scale factors corresponding to de-coherence $\dot{R}_{PT} = c$ and maximal expansion $\dot{R}_{max} = 0$ can be directly compared. From the Lemaitre equation

$$\frac{c^2}{R_{max}^2} = \frac{8\pi G_N}{3c^2} [\rho(R_{max}) + \rho_\Lambda] \cong \frac{8\pi G_N}{3c^2} \rho_{PT} \frac{R_{PT}^4}{R_{max}^4} \Rightarrow R_{max}^2 \cong 2R_{PT}^2. \quad (4.15)$$

Clearly, this closed system never expands much beyond the transition scale. Quite generally, the constraints of quantum measurability for quantized energy scales ($\dot{R}_\epsilon \leq c$) requires that all cosmologies which develop structure be spatially flat.

The evolution of the cosmology during the period for which the dark energy density is constant and de-coupled from the FL energy density is expected to be accurately modeled using the FL equations. There is a period of deceleration, followed by acceleration towards an approximately De Sitter expansion. The rate of scale parameter expansion is sub-luminal during a finite period of this evolution, as shown in Figure 3. The particular value for the scale at de-coherence (which is determined by the

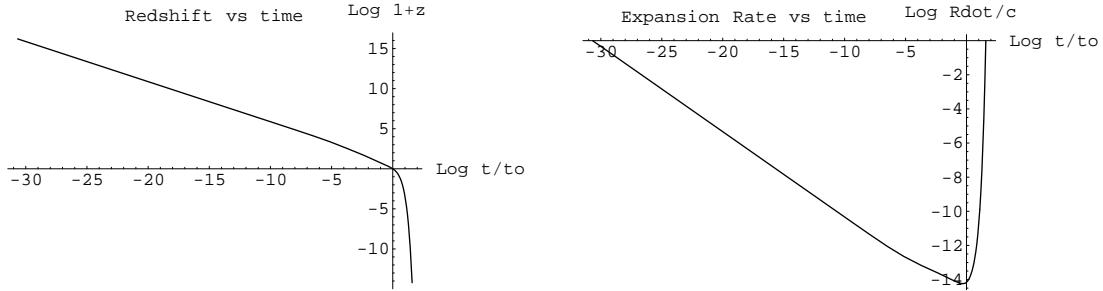


Figure 3: Graphs of redshift and expansion rate vs time

microscopic dynamics of the dark energy during de-coherence) chosen for the graphs is given in terms of the measured dark energy density $\rho_\Lambda = \epsilon^4$. The present time since the “beginning” of the expansion corresponds to the origin on both graphs. The value of the expansion rate is by assumption equal to the speed of light for any particular value chosen for R_{PT} , as well as when this expansion scale reaches the De Sitter radius R_Λ .

4.3 Estimate of size of source masses for the zero-point energy

To estimate the energy scales associated with the zero-point motions, assume that the dark energy is due to the uncertainty principle fluctuations of sources with

quantized energy units M (cf. discussion in Section 1 of Casimir effect beginning with Eq. 1.1). In general there are N such sources in a volume specified by R_ϵ^3 , and the quantized energy parameter ϵ . On average, each coherent energy unit contributes zero-point energy of the order

$$\frac{\epsilon}{N} \sim \frac{(\Delta P)^2}{2M} \geq \frac{\hbar^2}{8M(\Delta X)^2}, \quad (4.16)$$

where the uncertainty principle has been used in the form $(\Delta P)(\Delta X) \geq \frac{\hbar}{2}$. Replacing the spatial uncertainty with the coherence scale $\Delta X \sim R_\epsilon$ relates the energy scale to the zero-point energy $M \sim N\epsilon$. The cosmological density at the time of the phase transition is given by the ratio total (non-relativistic) energy to the volume of coherence

$$\rho_{PT} = \frac{NMc^2}{R_\epsilon^3} \sim N^2 \frac{\epsilon^4}{(\hbar c)^3} = N^2 \rho_\Lambda, \quad (4.17)$$

demonstrating that the cosmological density is related to the number of pairs of coherent energies $N(N - 1)/2$ undergoing zero-point motions. The FL equations determine the density during the phase transition from Eq. 4.9

$$\rho_{PT} \cong \frac{3}{8\pi} \frac{(M_P c^2 \epsilon^2)^2}{(\hbar c)^3} \quad (4.18)$$

giving direct estimates of the coherent energy units involved in the zero-point motions

$$N \sim \frac{M_P c^2}{\epsilon} \quad ; \quad M \sim M_P \quad (4.19)$$

That is, if the phase transition occurs in the radiation-dominated regime and the extrapolation back to it starts from a cosmological constant characterized by ϵ , the sources of the vacuum energy *must* be at the Plank mass scale, each with zero point energy $\sim \frac{\epsilon^2}{M_P c^2}$ on average, *independent* of the specific value of ϵ ! (Note also that the scale factor at the transition is nearly the same as the Schwartzschild radius for the mass contained within that radius.) This gives a clue as to why the dark energy gets frozen in. The coherent mass units undergoing zero-point behaviors have pairwise gravitational couplings $G_N M^2$ of order unity, whereas the de-coherent energy density during the FL expansion will consist of masses with considerably smaller gravitational couplings. For brevity, the collective modes of the Planck mass units

will here be referred to as gravons. The zero-point motions of those coherent energy units correspond to the vacuum energy of the gravons. It is here suggested that after the phase transition, this vacuum energy de-coheres from the FL energy density of the cosmology, which undergoes thermal expansion, and the microscopic coherence scale changes from R_ϵ to a much smaller scale given by the Compton wavelength of the energy scale m_{UV} associated with the critical density of the transition, as will be discussed in section 6. It is important to recognize that gravitational affects are strong due to quantum mechanics, not energy density, since strong gravitational coherence occurs at a scale much less than the Planck density.

As indicated in Eq. 3.1, a macroscopic scale R can be generated in terms of microscopic scales Δx and λ_m . In Fig. 2, the intuition is that a critical density of the cosmology must be reached before any coherent microscopic energy scale can “de-cohere” into distinct clusters. Prior to reaching this density, there is no available “space” to de-cohere into. Since the previous discussion indicates that many mass units are still within the coherence distance during this phase transition, one suspects a macroscopic quantum system during this time. Any microscopic mass m with the coherence scale defined in Eq. 3.1 will have zero-point energies of the order $\frac{(\Delta P)^2}{2m} \sim \epsilon$ at the time of the phase transition, which will red shift as the cosmology expands. The specific equation of state depends on the details of the macroscopic quantum system. For a gravon gas with energy $(N_g + \frac{1}{2})\hbar k_g c = (2N_g + 1)\epsilon$, the exponent in Eq. 2.2 has the value $b = 1$.

4.4 Holographic considerations

The entropy of the system during the phase transition period will be examined to check that expected holographic bounds are not exceeded. The Fleisher-Susskind[24] entropy limit considers a black hole as the most dense cosmological object of a given size, limiting the entropy according to

$$S \leq S_{hole}^{black} = \frac{k_B c^3}{\hbar} \frac{A}{4G_N}, \quad (4.20)$$

where A is a light-like bounding area. For a radiation dominated cosmology at de-coherence, the entropy is proportional to the number of quanta, and is related to the

energy density using $\left(\frac{c}{R_\epsilon}\right)^2 \cong \frac{8\pi G_N}{3} \rho_{PT}$ by

$$\frac{S}{V} = \frac{4}{3} \frac{\rho_{PT}}{T_\epsilon} \Rightarrow S = \frac{4}{\pi G_N} \frac{R_\epsilon}{T_\epsilon}. \quad (4.21)$$

Examining this for the instantaneously light-like area $\dot{R}_\epsilon = c$ given by $A = 4\pi R_\epsilon^2$ the ratio of the entropy in a thermal environment to the limiting entropy during thermalization is given by

$$\frac{S}{A/4G_N} \cong \frac{4}{\pi^2} \left(\frac{1}{R_\epsilon T_\epsilon} \right) \sim 10^{-16}. \quad (4.22)$$

Clearly this result satisfies the FS entropy bound period of the phase transition.

5 Scenarios Prior to Phase Transition

Using quantum measurability arguments, it has been shown that any quantized energy scale E cannot satisfy uncertainty relations prior to the time that the cosmological scale associated with that energy $R_E = \frac{\hbar c}{E}$ has a subluminal expansion rate $\dot{R}_E \leq c$. The latest quantized energy scale which manifests is associated with the IR behaviors of the system. For times prior to when the UV mode of the system can satisfy quantum measurability requirements, the behavior of the system is expected to be anomalous. Possible scenarios for earlier states of the universe which could connect to the description presented above will be examined in this section. The results involving the amplitude of CMB fluctuations and relative size of dark energy to FL energy are independent of the physical scenario prior to the luminal expansion rate of the scale associated with the dark energy. Therefore, the ideas discussed in this paper cannot be used to distinguish between these scenarios.

5.1 Inflationary prior state

If both general relativity and quantum mechanics can be reliably used to describe the cosmology prior to the time of the decoupling of the dark energy, quantum measurability constraints on the gravitational interactions (which have couplings of order unity) require a change in the state of the energy density. One possible scenario would

demand that this energy density be in the form of vacuum energy with respect to the forms of matter prevalent in the cosmology shortly after the transition.

Assuming a transition which conserves energy, if the energy density of the present cosmology during the phase transition is set by the inflationary energy density, the DeSitter scale of the inflation Λ_i is defined in terms of the dark energy scale of the present cosmology using

$$\rho_{\Lambda_i} = \frac{\Lambda_i c^3}{8\pi G_N} = \frac{3c^2}{8\pi} \frac{(M_P c^2)^2 \epsilon^2}{(\hbar c)^3} \Rightarrow \Lambda_i = \frac{3}{R_\epsilon^2}. \quad (5.1)$$

The entropy of the DeSitter horizon associated with this early inflationary cosmology satisfies

$$S_\epsilon = \frac{A_\epsilon}{4G_N} = \pi \left(\frac{M_P c^2}{\epsilon} \right)^2 k_B, \quad (5.2)$$

and the DeSitter temperature during the inflation is given by

$$k_B T_\epsilon = \frac{1}{2\pi R_\epsilon} = \frac{\epsilon}{2\pi}. \quad (5.3)$$

The scale of the horizon temperature during the inflation is comparable to the scale of the dark energy today. This is precisely the temperature scale that would drive thermal fluctuations of the same magnitude as the zero-point fluctuations that produce the dark energy during the phase transition to the post-inflationary cosmology of today.

Using the FL equation for the acceleration Eq. 4.10, the acceleration just after the phase transition is given by $\ddot{R}_\epsilon = -\epsilon$. The inflationary scale just prior to the phase transition has an acceleration given by $\ddot{R}_\epsilon = +\epsilon$. The transition requires a change in the scale acceleration rate of the order of the dark energy in each scale region of the subsequent decelerating cosmology.

5.2 Bjorken multiple inflations

It is of interest to consider a possible scenario first suggested by J. Bjorken[28, 29]. Since a luminal scale expansion rate plays a key role in the present paper, there is no reason not to expect prior and later periods of decelerations and inflations in this cosmology, as illustrated in Figure 4. When relevant scale expansion rate

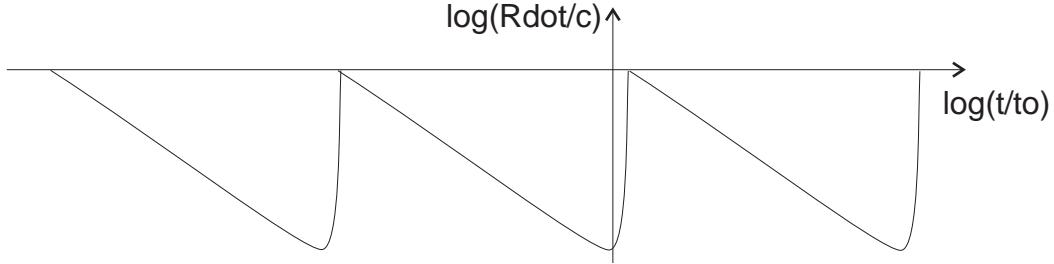


Figure 4: Multiple inflations limited by subliminal expansion rate

from previous expansion equals the speed of light, the next deceleration phase begins with the dark energy density of inflation driving the thermalized matter density of the next deceleration. In Bjorken's scenario, the landscape of the expansion need not scale uniformly as shown in Figure 4, i.e. valleys need not have self-similar depths and widths. The physics of any given deceleration/inflation cycle is determined by appropriate interaction energy scale of the microscopic dynamics. The cycles will maintain self-similarity only if all scales associated with the phase transitions and broken symmetries scale with the cosmology. If this is not the case, the next deceleration will have a temperature far too cold to produce any of the massive particles in the spectrum of the standard model.

Assuming that the vacuum density generated by the cosmological constant of the prior inflation becomes the FL density in Eq. 4.9 of new expansion, the phase transition relates the dark energy scales via

$$\rho_{\Lambda_i} \equiv \epsilon_j^4 = \frac{3}{8\pi} M_P^2 \epsilon_{j+1}^2 = \rho_{PT}, \quad (5.4)$$

where as before, the dark energy scale satisfies $\rho_{\Lambda} = \frac{\Lambda}{8\pi G_N} = \frac{3}{8\pi G_N R_{\Lambda}^2} = \frac{\epsilon^4}{(\hbar c)^3}$.

Considerable concern has been expressed in the literature concerning the Poincare recurrences associated with systems of finite entropy, such as in a DeSitter cosmology[30]. The entropy associated with a spherically symmetric horizon (while the physics of that horizon are valid) is given by $S_H = k_B \frac{\pi r_H^2}{L_P^2}$, and the temperature is given by $T_H = \frac{\hbar c}{2\pi k_B r_H}$. A finite entropy, regardless how large, indicates a finite number of possible configurations for the system, which will eventually recur into any “initial” state, eliminating historical imprints upon that system (along with any information

associated with those imprints). This return to a prior state has the same effective outcome as a “big crunch” in the re-ordering of the system. To explore whether the present scenario undergoes such recurrences with any likelihood, examine the entropy change during one of the transitions between inflation and deceleration:

$$\frac{S_{j+1}}{S_j} = \left(\frac{R_{\Lambda_{j+1}}}{R_{\Lambda_j}} \right)^2 = \frac{\rho_{\Lambda_j}}{\rho_{\Lambda_{j+1}}} = \frac{8\pi}{3} \frac{M_P^2}{\epsilon_{j+1}^2} \gg 1 \quad (5.5)$$

Therefore, since the entropy increases are relatively large, the time scales of recurrences should be examined to determine the likelihood of a recurrence prior to the onset of the following deceleration. The recurrences are expected to occur stochastically on time scales given by[30, 31]

$$t_{recurrence} \cong t_{reshuffle} e^{\frac{S_\Lambda}{k_B}}, \quad (5.6)$$

where $t_{reshuffle}$ is the typical time scale associated with the microscopic reshuffling of those configurations that give rise to the finite entropy S_Λ . One expects this reshuffling time (which is directly related to the mean free time, and is determined by microscopic dynamics) to be independent of the number of configurations (except perhaps through intensive variables like density, etc). The configurations are counted by the thermodynamic weight \mathcal{W} in the Boltzmann identification $S = k_B \log \mathcal{W}$. This reshuffling time can be estimated to be a fixed fraction f_R of the causal transit time $\sim \frac{R_\Lambda}{c}$ across the De Sitter patch (causal region), giving recurrence times of the order

$$t_{recurrence} \cong f_R \left(\frac{R_\Lambda}{c} \right) e^{\pi \left(\frac{R_\Lambda}{L_P} \right)^2} \cong f_R \left(\frac{R_\Lambda}{c} \right) e^{\frac{3}{8} \left(\frac{M_P c^2}{\epsilon} \right)^4}. \quad (5.7)$$

An estimate of the time within a given expansion phase can be estimated using the FL equation for a system with initial decelerating expansion of order n , followed by inflation due to the cosmological constant of the epoch:

$$\left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N}{3} \left[\rho_{PT} \left(\frac{R_{PT}}{R} \right)^n + \rho_\Lambda \right]. \quad (5.8)$$

The duration between times when $\dot{R} = c$ is about

$$\begin{aligned} c\Delta t &\cong (1 - \frac{2}{n}) R_\Lambda \log \frac{R_\Lambda}{R_{PT}} + \frac{2}{n} (R_\Lambda - R_{PT}) + \frac{1}{2} R_{PT} \\ &\sim R_\Lambda \log \frac{M_P c^2}{\epsilon} \end{aligned} \quad (5.9)$$

which is exponentially more rapid than the Poincare recurrence time. Thus, this scenario essentially eliminates Poincare recurrences, since the entropy is never bounded, and a recurrence during any cycle is extremely unlikely.

5.3 Quantum only cosmology

An alternative scenario connects the temporal progression in classical general relativity to gravitational de-coherence, implying a single coherent stationary quantum state of density ρ_{PT} prior to the phase transition. Since the state is not temporally defined, the FL equations do not describe the cosmological dynamics during this “pre-coherent” period. Temporal progression begins only when de-coherent interrelations break the stationarity of the quantum state, and is described by the Friedman-LeMaitre equations with appropriate scale eventually taking the value R_ϵ as the transition proceeds. In such a state, there is no initial temporal singularity, or well defined $t=0$. A quantum fluctuation of a size that produces dark energy ϵ with scale R_ϵ gives the same field dynamics as would an inflationary scenario with DeSitter scale R_ϵ that matches the phase transition into the decelerating epoch.

Other arguments can be made for expecting a substantial change in the thermal behavior of the cosmology in early times. As has been argued, the condition $\dot{R}_E = c$ gives an IR cutoff for microscopic energy scales that satisfy quantum measurability constraints. During earlier times, this cutoff eventually approaches any microscopic UV cutoff, limiting the number of available thermal states. As an illustration, a bosonic system has a density which satisfies $\rho = \rho_{IR} + \frac{4\pi}{(2\pi\hbar)^3} \int_{k_{IR}}^{k_{UV}} k^2 dk \frac{E(k)}{e^{\beta(E-\mu)} - 1}$. As the IR and UV cutoffs become comparable, the thermal behavior is expected to be significantly different from the usual behavior with negligible IR and large UV cutoffs. An initial state with complete occupation in a single mode would be expected to be stationary, and cannot be described with classical dynamics.

5.4 Classical relativistic prior state

One can examine a scenario for which the violation of the constraints of quantum measurability results in a classical only dynamics, described by the Friedman-

Lemaitre equations of general relativistic cosmology. Such a scenario is generally singular at $t = 0$. The question then arises whether any meaning can be given to a “radiation dominated” cosmology for a purely classical system. Since classical general relativity includes continuous distributions of matter described by an equation of state, there would seem to be no difficulty in *defining* such a cosmology in terms of the equation of state for pressure as one for which $P = (1/3)\rho$. If this scenario is to have ordinary electromagnetic radiation fields, they would be problematic if used to define a temperature because of the Rayleigh-Jeans ultraviolet divergence[32], associated with the statistics of classical modes instead of quantized (eg. Planck) modes. This precludes a statistical description of this scenario.

6 What is Special About ρ_{PT} in *This* Cosmology

Up to this point, any value for the critical density ρ_{PT} , and correspondingly for the dark energy scale ϵ , will generate the observed scale for the amplitude of fluctuations in the CMB radiation. The observed scale of dark energy might be just a random fluctuation. One might alternatively expect microscopic physics to fix the particular scale as a quantum phase transition associated with the UV energy scale for the gravitational modes, $\rho_{PT} \equiv (m_{UV}c^2)^4/(\hbar c)^3$. For instance, if the scale is associated with the density of thermal bosonic matter, the critical temperature is related to the (non-relativistic) mass by $k_B T_{crit} = \left(\frac{(2\pi)^2}{g_m \zeta(3/2) \Gamma(3/2)} \right)^{2/3} m_{UV}c^2 \cong \frac{6.6}{g_m^{2/3}} m_{UV}c^2$. Since this temperature is comparable to the ambient temperature of thermal standard model matter at this density, bosonic matter at this density would have a significant condensate component. Using the relation connecting microscopic and macroscopic scale given previously by $R_\epsilon \lambda_m \sim (\Delta x)^2$, the zero-point energies of each of the UV energy units m_{UV} is of the order of the dark energy $\frac{(\Delta P)^2}{2m_{UV}} \sim \epsilon$. Assuming that the cosmological vacuum energy is due to the zero-point motions of gravitating sources just at the availability of luminal equilibrations (consistent with quantum measurability), the dark energy density can be calculated from the coherence scale using

$$\rho_\Lambda = \frac{\epsilon}{R_\epsilon^3} = \frac{\epsilon^4}{(\hbar c)^3}. \quad (6.1)$$

This means that the cosmological constant gets fixed by the physical condition of a luminal expansion rate associated with a microscopic scale (and the associated de-coherence) being met.

The currently accepted values[6] for the cosmological parameters involving dark energy and matter will be used for the reverse time extrapolation from the present:

$$h_0 \cong 0.72; \quad \Omega_\Lambda \cong 0.73; \quad \Omega_M \cong 0.27. \quad (6.2)$$

Here h_0 is the normalized Hubble parameter. Note that this value implies that the universe currently has the critical energy density $\rho_c \simeq 5.5 \times 10^{-4} GeV \text{ cm}^{-3}$. The values of these parameters for the present cosmology are given by $\rho_\Lambda \simeq 4.0 \times 10^{-6} GeV/cm^3$, $R_\Lambda \simeq 1.5 \times 10^{28} cm \simeq 1.6 \times 10^{10} ly$, $\epsilon \simeq 2.35 \times 10^{-12} GeV$, $R_\epsilon \simeq 8.4 \times 10^{-3} cm$, $\rho_{PT} \simeq 1.3 \times 10^{55} Gev/cm^3$, $m_{UV} \simeq 3133 GeV$. If the cosmology were to remain a hot radiation dominated thermal system during the phase transition, with the microscopic degrees of freedom $g(T)$ due to the particle spectrum included, a temperature and redshift for the transition can be calculated:

$$\rho_{PT} \sim g(T_{PT}) \frac{\pi^2}{30} \frac{(k_B T_{PT})^4}{(\hbar c)^3} \Rightarrow k_B T_{PT} \sim 1300 GeV, z_{PT} \sim 1 \times 10^{16}. \quad (6.3)$$

6.1 A connection to microscopic physics

Choosing electro-weak symmetry restoration estimates of the early 1990's, Ed Jones[25, 26] predicted a cosmological constant with $\Omega_\Lambda \simeq 0.6$ before the idea of a non-vanishing small cosmological constant was fashionable. Motivated by this approach, examine symmetry breaking in the early universe [21]:

$$\mathcal{L} = \sqrt{-g} \left[-\frac{1}{2} (D_\mu \Phi_b) g_{\mu\nu} (D_\nu \Phi_b) + \frac{1}{4} m_\Phi^2 \Phi_b^2 - \frac{1}{8} f^2 \Phi_b^4 - \frac{1}{8} \frac{m_\Phi^4}{f^2} \right] + \mathcal{L}_{particle}, \quad (6.4)$$

where the particle Lagrangian includes the gauge boson field strength contribution $-F^{\mu\nu} F_{\mu\nu}/16\pi$ as well as any vacuum energy subtractions. In late times, the classical solution where the gauge fields vanish results in a non-vanishing vacuum expectation value for one of the field components

$$| < \Phi_1 > | = 0 \quad , \quad | < \Phi_2 > | = \frac{m_\Phi}{f} = \frac{m_B}{\tilde{e}}. \quad (6.5)$$

The constants have been arranged such that as the system settles into the phase transition, the expected FL energy density is correctly reproduced with zero contribution of the Higgs field to the cosmological energy density at late times. The strategy is to use the energy density due to the symmetry breaking (Higgs[33]) field prior to the thermalization of its excitations relative to the background given in Eq. 6.5 and the vector bosons into the microscopic particulate states whose remnants persist today. The action corresponding to this Lagrangian

$$W_{matter} \equiv \int \mathcal{L} d^4x \quad (6.6)$$

generate the conserved energy-momentum tensor in the Einstein equation

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta W_{matter}}{\delta g_{\mu\nu}}. \quad (6.7)$$

There is every indication that the cosmology will have extreme spatial homogeneity during the phase transition, so that for the present, spatial gradients will be neglected. For an FRW cosmology, the Jacobian factor can be calculated using $g = g_{00}g_{xx}g_{yy}g_{zz}$. The energy density can then be calculated as

$$T_{00} = \frac{1}{2}(\dot{\Phi} + \tilde{e}A_0\Phi)^2 - \frac{1}{4}m_\Phi^2\Phi_b^2 + \frac{1}{8}f^2\Phi_b^4 + \frac{1}{8}\frac{m_\Phi^4}{f^2} + T_{00}^{particle}. \quad (6.8)$$

When particulate degrees of freedom are negligible, the general temporal equation of motion for the field is given by

$$\frac{1}{R^3}\frac{d}{dt}\left(R^3[\dot{\Phi} + \tilde{e}A_0\Phi]\right) - \frac{1}{2}(m_\Phi^2 - f^2\Phi^2)\Phi = 0, \quad (6.9)$$

which has static solutions when Φ has vacuum expectation values of $\langle \Phi \rangle = 0$ and $|\langle \Phi \rangle| = \frac{m_\Phi}{f}$. In the earliest stages, when all fields are small, derivatives of the Φ field are seen to be of the order $H = \dot{R}/R \sim c/R_e = \epsilon$, which is expected to be considerably less than the rates associated with microscopic transitions involving the vector bosons. As the field initially evolves, the scalar potential is expected to have rates which depend on standard model couplings and mass ratios involving m_Φ with other particle masses, which should damp rapid changes in Φ , and smoothes the transition between the “static” solutions. The energy density of the system given by

T_{00} in Eq. 6.8 varies from $\frac{1}{8} \frac{m_\Phi^4}{f^2}$ for the symmetric solution which has no contribution from the particles to $\rho_{PT} + \rho_\Lambda$ when the symmetry is fully broken and the particles have become manifest. If the energy density is preserved during the transition, this gives a relation between the symmetry breaking parameters and the critical density of the cosmology:

$$\rho_{PT} \cong \frac{1}{8} \left(\frac{m_\Phi^2}{f} \right)^2, \quad (6.10)$$

(where ρ_Λ has been neglected) which can be solved to give

$$\begin{aligned} m_\Phi &= \frac{\tilde{e}}{m_B} \sqrt{8\rho_{PT}} \\ f &= \left(\frac{\tilde{e}}{m_B} \right)^2 \sqrt{8\rho_{PT}}. \end{aligned} \quad (6.11)$$

Substitution of the form of the expected density during the phase transition given in Eq. 4.9, the parameters are expected to satisfy

$$\begin{aligned} m_\Phi &= \sqrt{\frac{3}{\pi}} \frac{\tilde{e}}{m_B} M_P \epsilon \\ f &= \sqrt{\frac{3}{\pi}} \left(\frac{\tilde{e}}{m_B} \right)^2 M_P \epsilon. \end{aligned} \quad (6.12)$$

If the temporal progression defining the dynamics of the FL equations only begins after there is de-coherent energy density present, the initial stationary quantum state only becomes dynamical once the Higgs field begins to take a non-vanishing vacuum expectation value. However, if both general relativity and quantum properties hold in the earliest stages, the FL equations have an initial inflationary period with the scale parameter satisfying

$$\begin{aligned} \left(\frac{\dot{R}}{R} \right)^2 &= \frac{8\pi G_N}{3} \rho_{tot} \\ \rho_{tot} &\cong \left[\frac{m_\Phi^4}{8f^2} + \frac{1}{2}(\dot{\Phi} + \tilde{e}A_0\Phi)^2 + \frac{1}{8}f^2\Phi^4 - \frac{1}{4}m_\Phi^2\Phi^4 + \rho_{particles} \right] \end{aligned} \quad (6.13)$$

The dynamical equation relating the time derivatives of the component densities can be obtained from energy conservation $T^{0\mu}_{;\mu} = 0 = \dot{\rho}_{tot} + 3\frac{\dot{R}}{R}\rho_{tot}$. This equation describes the detailed thermalization of energy into the particulate states of present day cosmology.

6.2 Power spectrum considerations

A few remarks will be made concerning spatial inhomogeneities in the cosmology during and after the phase transition. Since the fluctuations form after any

inflationary periods, no fine tuning is necessary to prevent amplification of initial fluctuations[34]. The usual approach[35] involves examining metric perturbations $h_{\mu\nu}$ on the classical FRW metric $(ds^{FRW})^2 = -c^2 dt^2 + R^2(t) \mathbf{dx}^2 = R^2(t(\eta))(-d\eta^2 + \mathbf{dx}^2)$ in the form $g_{\mu\nu} = g_{\mu\nu}^{FRW} + h_{\mu\nu}$. Expressing the metric in terms of the conformal time η insures light cones of slope unity in these coordinates. The dynamics of the perturbations is most directly calculable using the form for the curvature tensor $R_{\mu\nu\rho\sigma} = \frac{1}{2}(g_{\mu\sigma,\nu\rho} + g_{\nu\rho,\mu\sigma} - g_{\mu\rho,\nu\sigma} - g_{\nu\sigma,\mu\rho}) + \frac{1}{2}g_{\lambda\alpha}(\Gamma_{\nu\rho}^\lambda\Gamma_{\mu\sigma}^\alpha - \Gamma_{\nu\sigma}^\lambda\Gamma_{\mu\rho}^\alpha)$. Expanding the small metric perturbations into momentum modes $h_{\mu\nu}(\eta, \mathbf{x}) \equiv \int d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{x}} \tilde{h}_{\mu\nu}(\eta, \mathbf{k})$, the dynamics are given by

$$\frac{d^2\tilde{h}_{\mu\nu}(\eta, \mathbf{k})}{d\eta^2} + 2\frac{R'(\eta)}{R(\eta)}\frac{d\tilde{h}_{\mu\nu}(\eta, \mathbf{k})}{d\eta} + k^2\tilde{h}_{\mu\nu}(\eta, \mathbf{k}) = 0 \quad (6.14)$$

The dynamics is directly solvable in terms of the reduced perturbations $\psi_{\mu\nu}(\eta, \mathbf{k}) \equiv R(\eta)\tilde{h}_{\mu\nu}(\eta, \mathbf{k})$, which satisfies

$$\frac{d^2\psi_{\mu\nu}(\eta, \mathbf{k})}{d\eta^2} + \left[k^2 - \frac{R''(\eta)}{R(\eta)}\right]\psi_{\mu\nu}(\eta, \mathbf{k}) = 0. \quad (6.15)$$

This means that short wavelengths that satisfy $k^2 >> \frac{R''(\eta)}{R(\eta)}$ will damp out with the scale factor $\tilde{h}_{\mu\nu}(\eta, \mathbf{k}) \cong \tilde{h}_{\mu\nu}(\eta_0, \mathbf{k})e^{|\mathbf{k}|(\eta-\eta_0)}/R(\eta)$, whereas long wavelength that satisfy $k^2 << \frac{R''(\eta)}{R(\eta)}$ will behave as $\tilde{h}_{\mu\nu}(\eta, \mathbf{k}) \cong \tilde{h}_{\mu\nu}(\eta_0, \mathbf{k}) + B_{\mu\nu}(\mathbf{k}) \int_{\eta_0}^{\eta} \frac{d\zeta}{R^2(\zeta)}$, which effectively freezes in those modes at their values at time η_0 . Since longer wavelengths satisfy quantum measurability constraints at later times, the relevant wavelengths should match those of the phase transition.

One expects most of the modes generated by the de-coherence which form the dark energy to evolve in the same manner as would those in an inflationary scenario for times after the horizon crosses the cosmological scale. However, UV modes are expected to differ significantly, due to microscopic de-coherent physics. Likewise, the IR modes (either horizon or DeSitter scale) could differ due to space-like coherence effects. Zero-point energies are important in the renormalization group descriptions of quantum fluids[38]. Other authors have found the IR behavior to be important[36, 37]. More detailed calculations on the power spectrum will be pursued in the future.

7 Conclusions and Discussion

This paper has presented evidence that current cosmological observations can be accounted for by the hypothesis that at early times there was a phase transition from a macroscopically coherent state which produced fluctuations consistent with those observed today. Gravitational de-coherence with fluctuations driven by zero-point motions gives the expected order for the amplitude of fluctuations in the CMB radiation, as well as the observed dark energy. Generally some form of cosmological quantum coherence (implying space-like correlations and phase coherence) is expected at densities far from Planck densities. Any macroscopic quantum system will manifest such correlations after it decays or dissolves into luminally disconnected regions. These conclusions are robust in the sense that any theory satisfying these constraints will produce the same magnitude of CMB fluctuations.

The constraints of quantum measurement (which for instance give the uncertainty principle) associates with any quantized energy scale a cosmological scale whose expansion rate is at most luminal. The phase transition of interest occurs when the dark energy scale satisfies this constraint. Spatial flatness is required for observed structure formation if one uses this relevant microscopic scale in the FL equations. Using the measured value for the dark energy, some form of microscopic manifestation of gravitational physics is expected on the TeV energy scale.

To relate the discussion of the general results to the observed value of the dark energy density, an argument has been presented to connect the critical density of the phase transition to the upper limit of quantized mass excitations given by the breaking of symmetry using the Higgs mechanism. In a ferromagnetic transition, one would expect excitations with energies comparable to the magnetization energy to destabilize that magnetization, restoring symmetry. Likewise, this critical density could represent the upper limit on the mass spectrum associated with the Higgs field. One should ultimately be able to motivate the form of the Higgs Lagrangian using density-density fluid dynamics in the early gravitating epoch. An exploration of the microscopic physics should give detail to the power spectrum of the fluctuations generated during the transition.

Since the results presented depend only on the physics of the transition, several prior scenarios have been explored which connect appropriately to the phase transition. Quantum coherence reproduces all of the expected results of an inflationary scenario except for magnetic monopole dilution. However, it should be recalled that the very reason for the development of the theory of special relativity was as an explanation of the covariance of electromagnetic fields in the absence of external magnetic sources. The lack of detection of magnetic monopoles could simply indicate their non-existence, and has not been taken as a motivating factor for this paper. Quite generally, the phase transition described above ties the top of the particulate mass spectrum associated with the transition to the observed value of the cosmological constant.

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